OCR
RECOGNISING ACHIEVEMENT

Monday 24 June 2013 – Afternoon

A2 GCE MATHEMATICS

4731/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4731/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

* 4 7 3 3 4 4 0 6 1 3 *

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A camshaft inside an engine is rotating with angular speed $42\,\text{rad s}^{-1}$. When the throttle is opened the camshaft speeds up with constant angular acceleration, and 8 seconds after the throttle was opened the angular speed is $76\,\text{rad s}^{-1}$.

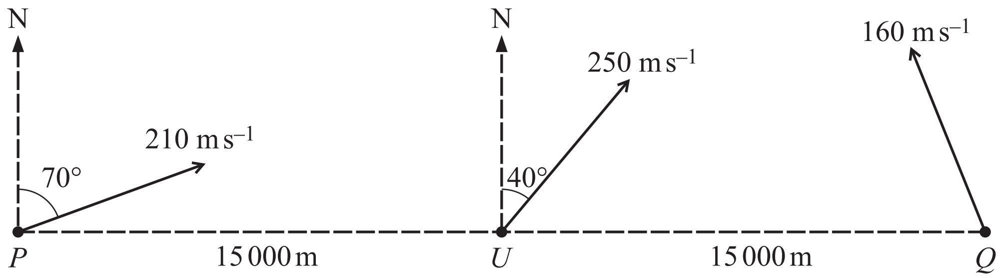
(i) Find the angular acceleration of the camshaft. **[2]**

(ii) Find the time taken for the camshaft to turn through 810 radians from the moment that the throttle was opened. **[3]**

2 A straight rod AB has length a. The rod has variable density, and at a distance x from A its mass per unit length is given by $k\left(4-\sqrt{\frac{x}{a}}\right)$, where k is a constant. Find the distance from A of the centre of mass of the rod. **[7]**

3 The region R is bounded by the x-axis, the y-axis, the curve $y = ae^{\frac{x}{a}}$ and the line $x = a\ln 2$ (where a is a positive constant). A uniform solid of revolution, of mass M, is formed by rotating R through 2π radians about the x-axis. Find, in terms of M and a, the moment of inertia about the x-axis of this solid of revolution. **[8]**

4



An unidentified aircraft U is flying horizontally with constant velocity $250\,\text{m s}^{-1}$ in the direction with bearing 040°. Two spotter planes P and Q are flying horizontally at the same height as U, and at one instant P is 15 000 m due west of U, and Q is 15 000 m due east of U (see diagram).

(i) Plane P is flying with constant velocity $210\,\text{m s}^{-1}$ in the direction with bearing 070°.

(a) Find the magnitude and bearing of the velocity of U relative to P. **[4]**

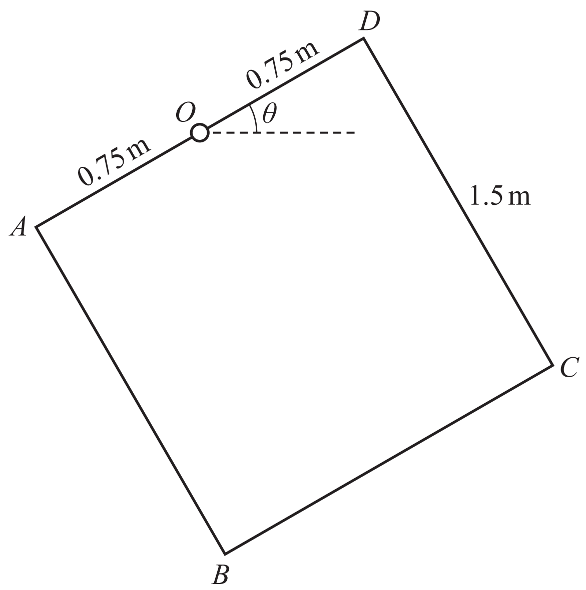
(b) Find the shortest distance between P and U in the subsequent motion. **[2]**

(ii) Plane Q is flying with constant velocity $160\,\text{m s}^{-1}$ in the direction which brings it as close as possible to U.

(a) Find the bearing of the direction in which Q is flying. **[4]**

(b) Find the shortest distance between Q and U in the subsequent motion. **[2]**

5



A square frame *ABCD* consists of four uniform rods *AB*, *BC*, *CD*, *DA*, rigidly joined at *A*, *B*, *C*, *D*. Each rod has mass 0.6 kg and length 1.5 m. The frame rotates freely in a vertical plane about a fixed horizontal axis passing through the mid-point *O* of *AD*. At time *t* seconds the angle between *AD* and the horizontal, measured anticlockwise, is θ radians (see diagram).

(i) Show that the moment of inertia of the frame about the axis through *O* is $3.15\,\text{kg}\,\text{m}^2$. **[4]**

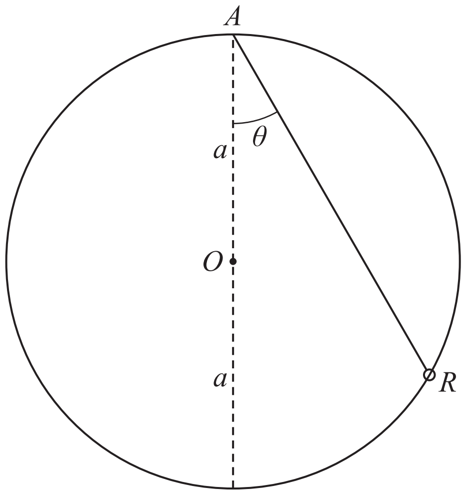
(ii) Show that $\dfrac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -5.6\sin\theta$. **[3]**

(iii) Deduce that the frame can make small oscillations which are approximately simple harmonic, and find the period of these oscillations. **[3]**

The frame is at rest with *AD* horizontal. A couple of constant moment 25 N m about the axis is then applied to the frame.

(iv) Find the angular speed of the frame when it has rotated through 1.2 radians. **[4]**

6



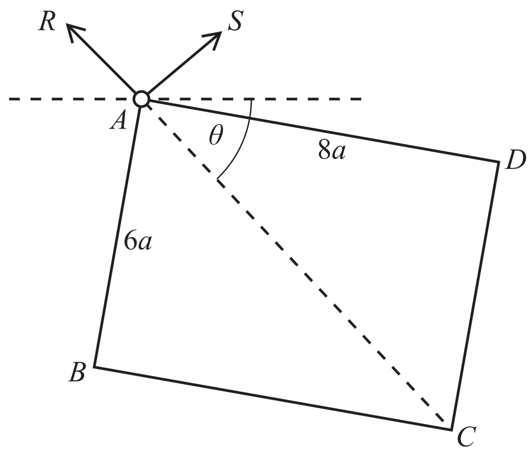
A smooth wire forms a circle with centre O and radius a, and is fixed in a vertical plane. The highest point on the wire is A. A small ring R of mass m moves along the wire. A light elastic string, with natural length $\frac{1}{2}a$ and modulus of elasticity $2mg$, has one end attached to A and the other end attached to R. The string AR makes an angle θ (measured anticlockwise) with the downward vertical (see diagram), and you may assume that the string does not become slack.

(i) Taking A as the reference level for gravitational potential energy, show that the total potential energy of the system is $mga(6\cos^2\theta - 4\cos\theta + \frac{1}{2})$. **[4]**

(ii) Show that there are two positions of equilibrium for which $0 \leqslant \theta < \frac{1}{2}\pi$. **[4]**

(iii) For each of these positions of equilibrium, determine whether it is stable or unstable. **[4]**

7



ABCD is a uniform rectangular lamina with mass m and sides $AB = 6a$ and $AD = 8a$. The lamina rotates freely in a vertical plane about a fixed horizontal axis passing through A, and it is released from rest in the position with D vertically above A. When the diagonal AC makes an angle θ below the horizontal, the force acting on the lamina at A has components R parallel to CA and S perpendicular to CA (see diagram).

(i) Find the moment of inertia of the lamina about the axis through A, in terms of m and a. **[3]**

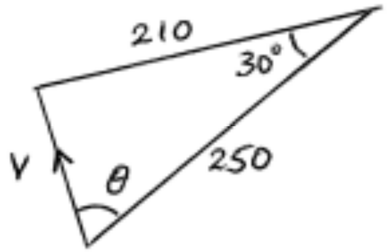
(ii) Show that the angular speed of the lamina is $\sqrt{\dfrac{3g(4+5\sin\theta)}{50a}}$. **[3]**

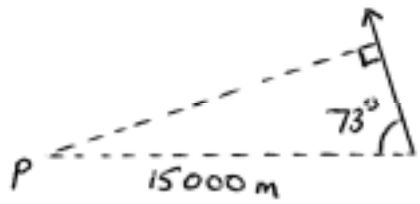
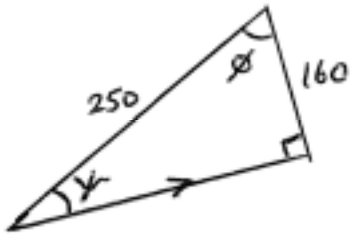
(iii) Find the angular acceleration of the lamina, in terms of a, g and θ. **[2]**

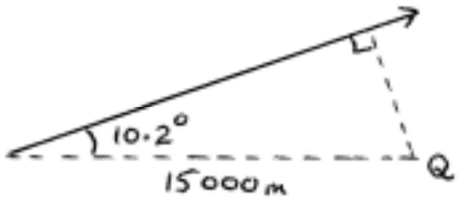
(iv) Find R and S, in terms of m, g and θ. **[6]**

Question			Answer	Marks	Guidance	
1	(i)		$76 = 42 + \alpha \times 8$	M1	Using $\omega_1 = \omega_0 + \alpha t$	
			Angular acceleration is 4.25 rad s^{-2}	A1		
				[2]		
1	(ii)			M1	Using $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	
			$810 = 42t + 2.125t^2$	A1	FT Quadratic equation for t	
			$t = \dfrac{-42 \pm \sqrt{42^2 + 4 \times 2.125 \times 810}}{2 \times 2.125}$			
		OR	$\omega_1{}^2 = 42^2 + 2 \times 4.25 \times 810$			
			$\omega_1 = 93$		M1 $\omega_1{}^2 = \omega_0{}^2 + 2\alpha\theta$ and $\omega_1 = \omega_0 + \alpha t$	Or equivalent
			$93 = 42 + 4.25t$		A1 FT Equation for t	
			Time is 12 s	A1		
				[3]		
2			$M = \displaystyle\int_0^a k\left(4 - \sqrt{\frac{x}{a}}\right)\mathrm{d}x$	M1	For $\displaystyle\int\left(4 - \sqrt{\frac{x}{a}}\right)\mathrm{d}x$	
			$= k\left[4x - \frac{2}{3}a^{-\frac{1}{2}}x^{\frac{3}{2}}\right]_0^a \quad (= \frac{10}{3}ka)$	A1	For $4x - \frac{2}{3}a^{-\frac{1}{2}}x^{\frac{3}{2}}$	
			$M\bar{x} = \displaystyle\int_0^a k\left(4 - \sqrt{\frac{x}{a}}\right)x\,\mathrm{d}x$	M1	For $\displaystyle\int\left(4 - \sqrt{\frac{x}{a}}\right)x\,\mathrm{d}x$	
			$= k\left[2x^2 - \frac{2}{5}a^{-\frac{1}{2}}x^{\frac{5}{2}}\right]_0^a \quad (= \frac{8}{5}ka^2)$	A2	For $2x^2 - \frac{2}{5}a^{-\frac{1}{2}}x^{\frac{5}{2}}$	Give A1 for one correct term
			$\bar{x} = \dfrac{\frac{8}{5}ka^2}{\frac{10}{3}ka}$	M1	Dependent on previous M1M1	
			$= \dfrac{12}{25}a = 0.48a$	A1		
				[7]		

Question			Answer	Marks	Guidance	
3			$M=\rho\int \pi y^2\mathrm{d}x=\int_0^{a\ln 2}\rho\pi a^2\mathrm{e}^{\frac{2x}{a}}\,\mathrm{d}x$	M1	For $\int\left(\mathrm{e}^{\frac{x}{a}}\right)^2\mathrm{d}x$	
			$=\left[\rho\pi\frac{a^3}{2}\mathrm{e}^{\frac{2x}{a}}\right]_0^{a\ln 2}=\frac{3}{2}\rho\pi a^3$	A1	For equation $M=\frac{3}{2}\rho\pi a^3$ oe	
			$I=\sum\frac{1}{2}(\rho\pi y^2\delta x)y^2=\frac{1}{2}\rho\pi\int y^4\mathrm{d}x$	M1	For $\int y^4\mathrm{d}x$	
			$=\int_0^{a\ln 2}\frac{1}{2}\rho\pi\left(a\mathrm{e}^{\frac{x}{a}}\right)^4\mathrm{d}x$	A1	Correct integral expression for I	
			$=\left[\frac{1}{2}\rho\pi\frac{a^5}{4}\mathrm{e}^{\frac{4x}{a}}\right]_0^{a\ln 2}$	M1	Integral is a multiple of $\mathrm{e}^{\frac{4x}{a}}$	
			$=\frac{15}{8}\rho\pi a^5$	A1		
			$=\frac{15}{8}\left(\frac{2M}{3\pi a^3}\right)\pi a^5$	M1	Obtaining I in terms of M and a	Dependent on first two M1M1
			$=\frac{5}{4}Ma^2$	A1	A0 for $\frac{\mathrm{e}^{4\ln 2}-1}{4(\mathrm{e}^{2\ln 2}-1)}Ma^2$ etc	*Accept* $\frac{15}{12}Ma^2$ *etc*
				[8]		

Question			Answer	Marks	Guidance	
4	(i)	(a)				
			$v^2 = 250^2 + 210^2 - 2\times 250\times 210\cos 30°$	M1	Equation for v	*Must be essentially correct* *M0 for* $\cos 150°$, $\cos 40°$ *etc*
			Magnitude is $125\ \text{m s}^{-1}$ (3 sf)	A1		
			$\dfrac{\sin\theta}{210} = \dfrac{\sin 30°}{125.2}$	M1	Equation for a relevant angle	*Use of sine rule and calculated side (less strict than previous M1)*
			$\theta = 57.0°$ Bearing is $343°$ (3 sf)	A1 **[4]**		
		OR	${}_U\mathbf{v}_P = \begin{pmatrix}250\sin 40° \\ 250\cos 40°\end{pmatrix} - \begin{pmatrix}210\sin 70° \\ 210\cos 70°\end{pmatrix}$		M1 Subtracting components	
			$= \begin{pmatrix}-36.64 \\ 119.7\end{pmatrix}$		A1	
			Magnitude is $125\ \text{m s}^{-1}$ Bearing is $343°$		M1 Finding magnitude or bearing A1 Both correct	

Question			Answer	Marks	Guidance	
4	(i)	(b)	As viewed from P  Shortest distance is $15000\sin 73°$ $= 14300$ m (3 sf)	M1 A1 **[2]**	Or other complete method for distance	M0 for $15000\cos 73°$
4	(ii)	(a)	 $\cos\phi = \frac{160}{250}$ $\phi = 50.2°$ Bearing is $350°$	B1 M1 A1 A1 **[4]**	Relative velocity perpendicular to $\mathbf{v}_Q$ Or $\psi = 39.8°$	

Question			Answer	Marks	Guidance	
4	(ii)	(b)	As viewed from Q  Shortest distance is $15000\sin 10.2°$ $= 2660$ m (3 sf)	M1 A1 **[2]**	Or other complete method for distance	
5	(i)		$I_{AD} = \frac{1}{3}(0.6)(0.75)^2 \quad (= 0.1125)$	B1		
			$I_{AB} = I_{CD} = 0.1125 + 0.6(0.75^2 + 0.75^2) \quad (= 0.7875)$ $I_{BC} = 0.1125 + (0.6)(1.5)^2 \quad (= 1.4625)$ $I = 0.1125 + 2 \times 0.7875 + 1.4625 = 3.15$	M1 M1 A1	 AG	M0 for $\frac{4}{3}(0.6)(0.75)^2 + (0.6)(0.75)^2$
		OR	$I = 4(0.1125 + 0.6 \times 0.75^2) + (2.4)(0.75)^2$ $= 1.8 + (2.4)(0.75)^2$ $= 3.15$		M1 for $0.1125 + (0.6)(0.75)^2$ M1 for $I_G + (2.4)(0.75)^2$ A1 AG	
				[4]		
5	(ii)		 $-2.4 \times 9.8 \times 0.75\sin\theta = 3.15\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$	B1 M1	For $2.4 \times 9.8 \times 0.75\sin\theta$ Equation of rotational motion	
		OR	$\frac{1}{2}I\omega^2 - mgh\cos\theta = K$ $I\omega\dot{\omega} + 2.4 \times 9.8 \times 0.75\sin\theta\,\dot{\theta} = 0$		M1 Differentiating energy equation A1	
			$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -5.6\sin\theta$	A1 **[3]**	AG	

Question			Answer	Marks	Guidance	
5	(iii)		When θ is small, $\sin\theta \approx \theta$	B1		
			$\frac{d^2\theta}{dt^2} \approx -5.6\theta$, which is (approx.) SHM	B1		
			Period ($\frac{2\pi}{\sqrt{5.6}}$) is 2.66 s (3 sf)	B1	*Accept* $\pi\sqrt{\frac{5}{7}}$ *etc*	Or $2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{3.15}{2.4\times 9.8\times 0.75}}$
				[3]		
5	(iv)		WD by couple is 25×1.2 $(=30)$	B1		
			Change in PE is $2.4\times 9.8(0.75-0.75\cos 1.2)$ $(=11.25)$	B1		
			$\frac{1}{2}(3.15)\omega^2 = 30-11.25$	M1	Equation involving KE, WD and PE	
			Angular speed is $3.45\ \text{rad s}^{-1}$ (3 sf)	A1		
				[4]		
6	(i)		GPE is $(-)mg(2a\cos\theta)\cos\theta$	B1	or $mg(a+a\cos 2\theta)$	
				M1	Using $\frac{\lambda x^2}{2l}$ (allow one error)	
			EPE is $\frac{2mg}{2(\frac{1}{2}a)}(2a\cos\theta-\frac{1}{2}a)^2$	A1		
			$V = 2mga(4\cos^2\theta - 2\cos\theta + \frac{1}{4}) - 2mga\cos^2\theta$			
			$= mga(6\cos^2\theta - 4\cos\theta + \frac{1}{2})$	A1	AG	
				[4]		
6	(ii)		$\frac{dV}{d\theta} = mga(-12\cos\theta\sin\theta + 4\sin\theta)$	B1	or $mga(-6\sin 2\theta + 4\sin\theta)$	Condone mga omitted, but penalise wrong sign
			Positions of equilibrium occur when $\frac{dV}{d\theta}=0$	M1		
			$\theta = 0$ and $\theta = \cos^{-1}\frac{1}{3}$ $(=1.23 \text{ or } 70.5°)$ (Hence two positions)	A1A1	*Can be awarded when B1 has not been given*	
				[4]		

Question			Answer	Marks	Guidance	
6	(iii)		$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mga(-12\cos^2\theta + 12\sin^2\theta + 4\cos\theta)$	B1	FT if comparable	or $mga(-12\cos 2\theta + 4\cos\theta)$
				M1	Considering the sign of $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}$	Or M2 (replacing B1M1) for another method to determine stability
			When $\theta = 0$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -8mga < 0$			
			so this position is unstable	A1	CWO	*www give BOD, but M1 (or M2) must be explicitly earned*
			When $\theta = \cos^{-1}\frac{1}{3}$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mga\left(-12\times\frac{1}{9} + 12\times\frac{8}{9} + 4\times\frac{1}{3}\right) = \frac{32}{3}mga > 0$			
			so this position is stable	A1 **[4]**	CWO	*As above*
7	(i)			B1	For $I_G = \frac{1}{3}m\left[(3a)^2 + (4a)^2\right]$	
			$I = \frac{1}{3}m\left[(3a)^2 + (4a)^2\right] + m(5a)^2$	M1	For $I_G + m(AG)^2$	
		OR			B1 For $I_{AD} = \frac{4}{3}m(3a)^2, I_{AB} = \frac{4}{3}m(4a)^2$	
			$I = \frac{4}{3}m(3a)^2 + \frac{4}{3}m(4a)^2$		M1 For $I_{AD} + I_{AB}$	
			$I = \frac{100}{3}ma^2$	A1 **[3]**		
7	(ii)		$\frac{1}{2}I\omega^2 = mg(4a + 5a\sin\theta)$	M1	Equation involving KE and PE	
			$\frac{50}{3}ma^2\omega^2 = mga(4 + 5\sin\theta)$	A1	FT	
			Angular speed $\omega = \sqrt{\frac{3g(4+5\sin\theta)}{50a}}$	A1 **[3]**	AG	

Question			Answer	Marks	Guidance	
7	(iii)		$mg(5a\cos\theta) = I\alpha$	M1	Equation of rotational motion	Or differentiating energy equation Or writing $\omega\dfrac{d\omega}{d\theta}$ in terms of θ
			Angular acceleration is $\dfrac{3g\cos\theta}{20a}$	A1 **[2]**	*Accept* $\dfrac{15g\cos\theta}{100a}$ *etc*	
7	(iv)			M1	For radial acceleration $r\omega^2$	
			$R - mg\sin\theta = m(5a)\omega^2$ $R - mg\sin\theta = \dfrac{3}{10}mg(4+5\sin\theta)$	A1		
			$R = \dfrac{1}{10}mg(12+25\sin\theta)$	A1		
				M1	For transverse acceleration $r\alpha$	Or use of $I_G\alpha$
			$mg\cos\theta - S = m(5a)\alpha$ $mg\cos\theta - S = \dfrac{3}{4}mg\cos\theta$	A1		Or $S(5a) = (\frac{25}{3}ma^2)\alpha$
			$S = \dfrac{1}{4}mg\cos\theta$	A1 **[6]**		